

# AN INNOVATIVE STRUCTURAL MODE SELECTION METHODOLOGY: APPLICATION FOR THE X-33 LAUNCH VEHICLE FINITE ELEMENT MODEL

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## Abstract

An innovative methodology for determining structural target mode selection and mode selection based on a specific criterion is presented. An effective approach to single out modes which interact with specific locations on a structure has been developed for the X-33 Launch Vehicle Finite Element Model (FEM). The presented Root-Sum-Square (RSS) displacement method computes resultant modal displacement for each mode at selected degrees of freedom (DOF) and sorts to locate modes with highest values. This method was used to determine modes, which most influenced specific locations/points on the X-33 flight vehicle such as avionics control components, aero-surface control actuators, propellant valve and engine points for use in flight control stability analysis and for flight POGO stability analysis. Additionally, the modal RSS method allows for primary or global target vehicle modes to also be identified in an accurate and efficient manner.

## Introduction

The X-33 Vehicle is a half scale prototype vehicle that will demonstrate in flight new technologies needed for a Reusable Launch Vehicle (RLV). The industry team includes Lockheed Martin (Project Lead), AlliedSignal Aerospace, B.F. Goodrich (previously Rohr), Rocketdyne, and nearly all the NASA centers. The X-33 lifting body consist of a aluminum lithium liquid oxygen tank sitting on top of a pair of composite liquid hydrogen tanks, (see figure 1). Two aerospike engines, which will provide 410,000 lbs. of thrust, sits behind a

thrust structure. Canted fins, vertical fin, body flap, and reaction control (thrust) systems provide vehicle flight control.

The X-33 Vehicle FEM is also a cooperative effort between industry and government. Rocketdyne was responsible for the Aerospike Engine FEM. Lockheed-Martin built the vehicle's thrust structure, landing gears, intertank, lox feedline, LH2 and lox propellant tank FEM's. B.F. Goodrich provided FEM of the canted fin control surfaces. Marshall/NASA modeled the body flap and vertical fin control surfaces, the thermo protection system, and was also responsible for the overall X-33 FEM integration.

Traditionally, determining dominate or target modes typically includes visual inspection of the mode shape deformed plots, inconjucion with analytical techniques such as modal effective mass and reaction evaluation, kinetic energy, group energy, element strain energy and drive point residue methods. Two techniques are being utilized for the X-33 FEM effort; modal kinetic energy and the modal Root-Sum-Square (RSS) displacement method. Results are verified by agreement between the two methods and visual inspection of selected modes and certain omitted modes.

Over the course of three years, five vehicle design iteration cycles have taken place. Between that time the vehicle X-33 FEM grew from approximately 80,000 DOF to a maximum of 176,000 DOF. After a concerted effort to reduce the model size, the DOF count currently stands at 122,000. As such the modal density is extremely high, producing nearly 1,000 highly coupled modes over a frequency range of 0 to 55 Hz for one flight or propellant tank fill condition. A bandwidth of .05 Hz between modes is typical for this model, resulting in clusters or families of similar behaving modes. The last design cycle included ten flight trajectory tank fill conditions (0 to 55 Hz), eight POGO vehicle tank fill conditions (0 to 35 Hz), three Ground Vibration Test (modal survey) tank fill conditions (0 to 55 Hz) and several other special structural vehicle dynamics investigation runs.

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Approximately over 22,000 vehicle modes were required to be sorted based on different modal criteria solely for the last design cycle.

The X-33's POGO stability analysis required the identification of the highest modal gain along the longitudinal or axial direction. Whereabouts, the controls stability analysis required the identification of the highest modal gain modes regardless of direction, i.e. a function of three-dimensional space. A method was needed to sort accurately and quickly through enormous amounts of data and extract the required modes of interest. The modal RSS displacement method was developed for this purpose. The RSS methodology's accuracy has been verified over a two-year period reviewing thousands of X-33 FEM modes by Marshall/NASA engineers. The results have been excellent.

### **RSS Methodology/Results**

The vibratory dynamic equation of motion for an undamped free vibration system is:

$$[m]\{\ddot{x}\} + [k]\{x\} = 0 \quad (\text{Eq. 1})$$

Where  $[m]$  is the mass matrix  
 $\{\ddot{x}\}$  is the acceleration matrix  
 $[k]$  is the stiffness matrix  
 $\{x\}$  is the displacement matrix

This equation yields a set of roots or eigenvalues, which in turn determine a set of natural mode shapes or eigenvector coefficients. Because the characteristic equations used to solve equation 1 are homogenous, there is not a unique solution for the eigenvectors coefficients and therefore we can only obtain a ratio among the eigenvector coefficients. The mode shape is defined by the ratio of the amplitudes of motion at the various points on the structure when excited at its natural frequency. If one of the elements of the eigenvector is assigned a certain value, the rest of the elements are also fixed because the ratio between any two elements is constant. The RSS takes advantage of this fact. Otrhonormal modes are used the RSS method. Normalization to a unit value of the largest eigenvector displacement is applied to the entire model (all the DOF) for all the modes.

Now, not only can a direct comparison between modes for a particular location can take place, but a comparison between modes from different tank fill conditions (flight configurations) can also take place for a particular location. For example, modes of interest can be identified by visually noting the degree of modal

displacement or deformation at a certain location on the structure; i.e. noting the vehicle modes in which the canted fin actually distorted identifies canted fin modes. The RSS computes magnitude resultant modal displacement values for each mode at selected degrees of freedom (DOF) and sorts to locate modes with highest values.

$$(\text{RSS Resultant Value})_j = \sum [\phi_x^2 + \phi_y^2 + \phi_z^2]^{1/2} \quad (\text{Eq. 2})$$

Where:  $(\text{RSS})_j$  is the summation magnitude value for mode j  
 $\phi$ 's are the eigenvector translation coefficients for mode j for the selected nodes of interest

The modes with the highest RSS displacement values have the highest overall motion and energy for that particular location or nodes. For the canted fin example, several hard points could be chosen along the span and chord and spatially spaced such that expected mode shapes be reasonably covered. Then keying on the RSS displacement values of these nodes, literally thousand of non-interest modes are easily and effectively filtered out, leaving only modes of interest. It has been shown that four points can pick up the canted fin modes from the thousands X-33 FEM vehicle modes. As for vehicle target modes, it has been shown that 86 nodes (out of 22,000 FEM nodes) can identify all the vehicle modes within a 0-25 Hz band. Above this range for the X-33 FEM, all target modes were aero-control surface modes, which were RSS individually.

Additionally, the RSS written software also sums the absolute modal displacement in the three axis directions for each mode:

$$R_{xj} = \sum |\phi_x|, R_{yj} = \sum |\phi_y|, R_{zj} = \sum |\phi_z| \quad (\text{Eq. 3})$$

Where:  $R_j$  is the absolute summation value for mode j in the x, y, and z component direction

Again, the RSS routine then sorts to locate modes with highest values. This yields inside to the directional modal dynamic behavior of the modes. In this fashion looking at modes with the highest  $R_x$  summation values, identified longitudinal or POGO modes. In the canted fin example, the directional summation ( $R_x$ ,  $R_y$ , &  $R_z$ ) values would give insight as to which bending mode was being identified by resultant RSS displacement value.

The RSS displacement method is particularly efficient when used with NASTRAN. The EIGRL card can be

set to MAX, which automatically normalizes the unit value of the largest displacement in the analysis set. The whole model needs to be run initially saving the master and dball files. A restart using the Case Control Set command can be used to punch out the eigenvector coefficients for the nodes of interest. A FORTRAN routine was written to read in the NASTRAN punch eigenvector file data, and to sum and sort the RSS displacement values for each mode.

The Table 1 indicates the vehicle target modes for the upcoming X-33 GVT based on the design cycle 4 FEM for the three tank fill configurations chosen by Lockheed and NASA engineers. Approximately three thousand modes needed to be sorted to determine modes of interest. Table 2 is an example of the RSS displacement search printout of the vehicle target modes for the GVT full tank configuration. Utilizing a "threshold RSS limit value", non-interest modes (i.e. modes with RSS values less than the threshold value) are not printed out. Using only eighty-six key nodes, the RSS displacement method successfully picked out every primary vehicle mode from 0 to 27 Hz, (or 27 modes out of 35 modes primary modes from a possible mode set of 953). The modes missed are due to the shear number of "other vehicle key nodes" masking the aero-control surface modes. In other words, all the aero-control surface modes can be readily identified by the RSS displacement method if the key nodes were only located on those components, an individual RSS component run. The other contributing factor in this case, is that the vehicle FEM modes became extremely difficult to identify at the higher modes because the coupling effect became more pronounced at these frequencies.

The results presented for this paper were for a large structure, the X-33 FEM. However, the RSS displacement method is best suited to extract the modes of interest for a localized area, such as a vertical fin, or avionics bay, etc. These localized areas only need a few key RSS nodes to be identified and will not run the risk of being averaged out by a large number of RSS nodes.

#### A Note Using NASTRAN and RSS Method

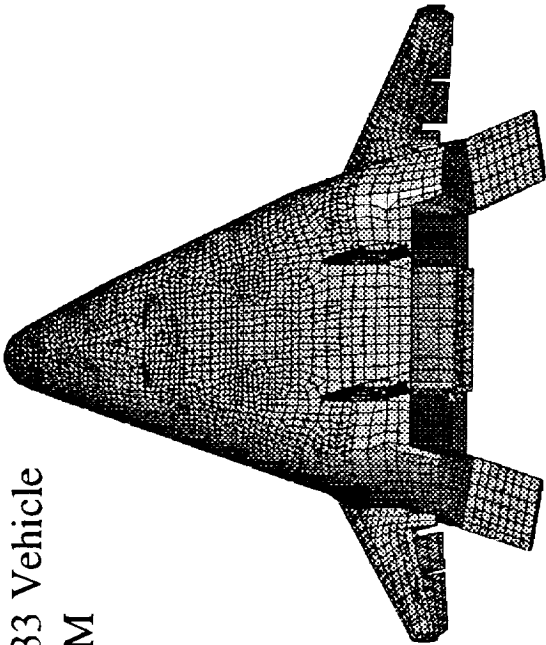
Strictly speaking, the RSS method requires that the resultant magnitude (Eq.2) value for each node be calculated and then for each mode be normalized with respect to the largest resultant magnitude value in that set. However, NASTRAN normalizes with respect to the largest eigenvector component for each mode. This means that the maximum resultant magnitude will be between unity and the square root of three or a value of

1.73 when using the MAX setting in NASTRAN. These are extreme values and have not been observed to occur. The experience is that when using NASTRAN MAX option only, the average maximum resultant value for any mode appears to be about ~1.3. More importantly, the RSS method consistently sorts the highest modes the highest, and the lowest modes to the lowest ranking when compared between component and resultant normalization. The error manifests itself by switching mode rank order in the mid-mode ranking range. This is easily overcome, by lowering the threshold to include a few modes of non-interest assuring all interest modes are identified.

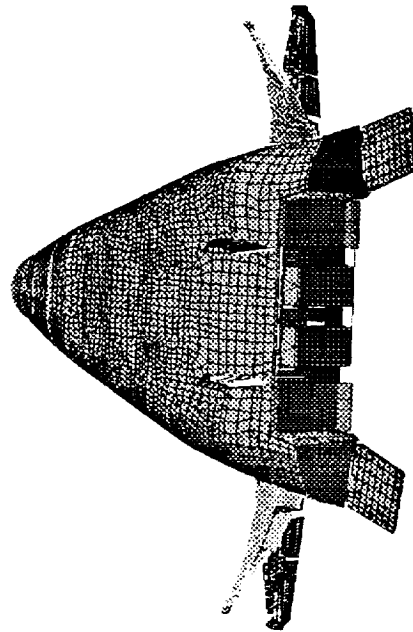
#### Conclusion

An innovated structural mode selection methodology has been presented as used for the X-33 Launch Vehicle FEM. The Root-Sum-Square displacement methodology has been used for the past two years and verified by comparison to several other standard mode selection method results and through visualization of the modes by both government and contractor engineers. The results have been good. The RSS displacement method is more accurate for localized areas, but can be applied to large structures if good engineering judgement is used in picking RSS key nodes. The method is a straight forward to implement, especially with conjunction with NASTRAN.

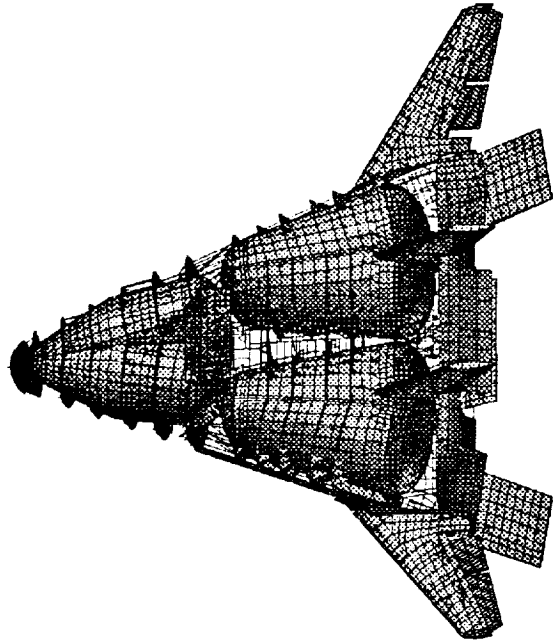
X-33 Vehicle  
FEM



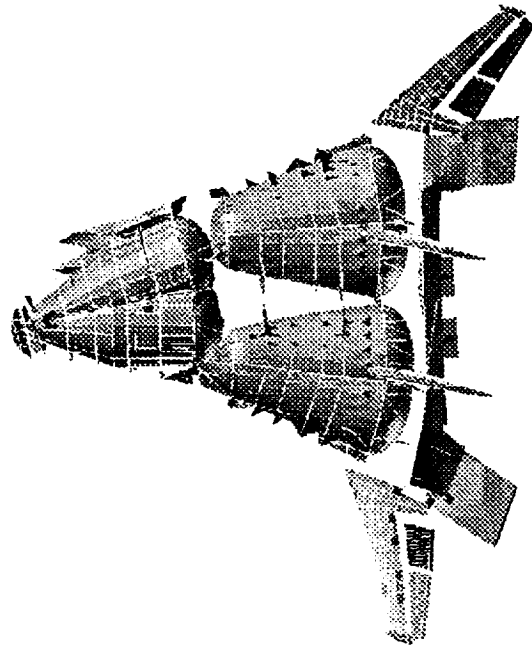
X-33 FEM with TPS



Canted Fin Symmetric Bending (6.41 Hz)



X-33 FEM Cut-away



Vehicle Yaw (11.0 Hz)

Figure 1

**GVT Target Modes For X33 Full, Partial, & Empty Vehicle  
LN2 Propellant and Isolation System (Load Cycle 4)**

11/3/98 LMSW/MSFC

	FULL		PARTIAL		EMPTY		Description
	Class- mode #	Freq. Hz	Class- mode #	Freq. Hz	Class- mode #	Freq. Hz	
1	S-1	0.18	S-1	0.20	S-1	0.30	vehicle/suspension pitch mode about base
2	S-2	0.36	S-2	0.41	S-2	0.57	vehicle/suspension yaw mode about base
3	S-6	1.49	S-4	1.44	S-3	1.28	vehicle/suspension roll mode
4	S-4	1.37	S-3	1.42	S-4	1.62	vehicle/suspension axial mode
5	S-8	2.25	S-6	2.08	S-5	1.73	vehicle/suspension lateral (yaw about lox tank)
6	S-9	2.37	S-7	2.28	S-6	1.94	vehicle/suspension normal (pitch about lox)
7	P-13	5.53 A	P-10	5.90 A	-	-	Vehicle/Lox Tank Torsion Mode // Partial + canted fin anti-sym
8	P-15	6.40	P-11	6.42	P-7	6.41	canted fin symmetric bending
9	P-16	6.46	P-13	7.32	P-9	6.91	canted fin anti-symmetric bending
10	P-18	7.77 B	-	-	-	-	LOX tank Z Bending/ Vehicle Z Bending
11	P-19	7.84	P-14	7.86	-	-	LOX Tank Y Bending + Vehicle Twisting about Base
12	-	-	-	-	P-10	8.32 A	Vehicle Torsion + Lox Frame 6&7
13	-	-	P-17	8.67 B	-	-	LOX Tank Z Bending: Vehicle Z Bending
14	P-23	8.85 C	-	-	-	-	LOX Tank Y Bending Mode
15	-	-	P-18	9.06 C	-	-	LOX Tank Y + Frame Lox 6&7
16	S-24	9.26 E	-	-	-	-	LOX Tank Twisting + Frames 6&7 + Canted Fin + Body Flap
17	-	-	P-19	9.20 E	-	-	LOX Tank Yaw w/ some Twist + Vehicle Twisting about Base
18	P-32	10.59	P-27	10.60	P-18	10.44	body flap symmetric
19	P-35	11.05	P-29	11.12	P-17	10.20	body flap anti-symmetric
20	-	-	-	-	P-21	10.90 B	Vehicle Normal (Z) Bending + Avionics Bay + Body Flap
21	-	-	-	-	P-22	11.00 C	vehicle yaw (LOX tank)
22	-	-	-	-	S-27	11.42	vehicle yaw
23	P-40	11.33	P-35	11.57	-	-	canted fin In-plane : anti-symmetric
24	-	-	-	-	P-28	11.88 E	vehicle yaw + frame 6&7 + Canted fin
25	-	-	-	-	S-29	11.99	canted fin In-plane : symmetric
26	P-42	11.61	P-36	11.92	-	-	Vehicle Pogo mode and canted fin Inplane symmetric bending
27	P-45	12.29 F	-	-	-	-	Vehicle Pogo model / Axial Mode ((Cf In Plane w/ Vehicle))
28	P-82	15.49	P-72	15.48	P-52	14.14	Vehicle Z Bending
29	-	-	P-78	16.43 F	-	-	Pogo Axial Vehicle mode
30	P-92	16.74	P-84	16.75	P-74	16.75	vertical fin anti-symmetric
31	P-94	16.92	P-86	16.94	P-77	16.97	vertical fin symmetric
32	P-97	17.20	P-92	17.84	-	-	Lox Tank Bulge Mode ((w/ Lox Frame axial motion))

Table 1

**GVT Target Modes For X33 Full, Partial, & Empty Vehicle  
LN2 Propellant and Isolation System**

11/3/98 LMSW/MSFC

33	P-102	17.85	P-92	17.84	P-81	17.84	canted fin torsion / outboard elevon symmetric
34	P-107	18.31	P-96	18.33	P-87	18.35	canted fin torsion/ outboard elevon anti-symmetric
35	-	-	-	-	P-89	18.54	inboard elevon symmetric
36	-	-	-	-	P-90	18.58	inboard elevon anti-symmetric
37	S-116	18.81	S-107	19.20	-	-	Vehicle 3rd Bending coupled mode
38	P-274	26.96	P-264	26.97	P-247	26.90	canted fin 2nd bending out of phase
39	P-277	27.13	P-266	27.11	P-249	27.00	canted fin 2nd bending in phase
40	P-279	27.31	P-269	27.32	P-252	27.33	vertical fin rudder anti-symmetric
41	P-282	27.45	P-272	27.45	P-256	27.45	vertical fin rudder symmetric
42	S-407	32.08	S-402	32.31	S-380	32.24	vertical fin in-plane
43	P-485	34.62	P-459	34.20	P-436	34.16	body flap torsion / symmetric
44	P-477	34.33	P-469	34.57	P-447	34.53	body flap torsion / anti-symmetric
45	S-628	41.63	S-597	40.82	S-571	40.81	canted fin 3rd bending / avionics bay
46	P-836	51.60	P-819	51.63	P-773	51.59	vertical fin torsion/anti-symmetric
47	P-840	51.81	P-821	51.72	P-777	51.87	vertical fin torsion/symmetric

**SUMMARY**

<b>FULL</b>	<b>PARTIAL</b>	<b>EMPTY</b>
953 Total FEM Modes	886 Total FEM Modes	~800 Total FEM Modes
35 Total GVT Target Modes	35 Total Target Modes	33 Total GVT Target Modes
25 Primary Target Modes	26 Primary Target Modes	23 Primary Target Modes
10 Secondary Modes	9 Secondary Modes	10 Secondary Modes

Table 1 (con't)

# RSS Displacement Search Results Vehicle Full\_GVT

NASA Struct Reduced FEM/ X33 Model c4/ Pretest Search  
Fill\_GVT\_In

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Freq= 0.E+0 to 55.

threshold= 1.5    #modes = 953    #dof= 258    ( ) Notes for AIAA Paper  
(or 86 Nodes in RSS Search)

Resulant Motion of RSS Nodes =  $(x^2+y^2+z^2)^{.5}$

	mode #	RSS Value	Freq. (Hz)	
1	1	24.2969	0.1808**	(Target Mode Visually ID: S-1)
2	2	31.2164	0.3646**	(Target Mode Visually ID: S-2)
3	3	3.1378	0.5696*	
4	4	82.8068	1.3741**	(Target Mode Visually ID: S-4)
5	5	8.3979	1.4554**	
6	6	24.8577	1.4914**	(Target Mode Visually ID: S-6)
7	7	13.0525	2.1408**	
8	8	18.9139	2.2473**	(Target Mode Visually ID: S-8)
9	9	42.6869	2.3667**	(Target Mode Visually ID: S-9)
10	11	2.0993	4.1329*	
11	13	3.4374	5.5324*	(Target Mode Visually ID: P-13)
12	15	3.5313	6.3970**	(Target Mode Visually ID: P-15)
13	16	4.9361	6.4581**	(Target Mode Visually ID: P-16)
14	18	16.6601	7.7719**	(Target Mode Visually ID: P-18)
15	19	8.1653	7.8355**	(Target Mode Visually ID: P-19)
16	20	2.2717	8.3583*	
17	22	2.5715	8.5490*	
18	23	5.0881	8.8501**	(Target Mode Visually ID: P-23)
19	24	7.1279	9.2583**	(Target Mode Visually ID: S-24)
20	30	3.1517	10.3370*	
21	32	3.1954	10.5886*	(Target Mode Visually ID: P-32)
22	35	4.0228	11.0533**	(Target Mode Visually ID: P-35)
23	38	1.9874	11.2734	
24	39	2.2565	11.2803*	
25	40	3.6870	11.3322**	(Target Mode Visually ID: P-40)
26	42	10.8559	11.6068**	(Target Mode Visually ID: P-24)
27	43	1.9004	11.6629	
28	45	5.5866	12.2860**	(Target Mode Visually ID: P-45)
29	72	1.6075	14.3467	
30	82	5.6439	15.4933**	(Target Mode Visually ID: P-82)
31	84	2.1897	15.7687*	
32	92	2.1649	16.7441*	(Target Mode Visually ID: P-92)
33	94	2.5973	16.9149*	(Target Mode Visually ID: P-94)
34	95	1.6982	16.9508	
35	96	2.7638	17.0105*	
36	97	2.9189	17.2026*	(Target Mode Visually ID: P-97)
37	99	3.4288	17.3587*	
38	101	3.9555	17.7632**	
39	102	2.3893	17.8474*	(Target Mode Visually ID: P-102)
40	107	2.1987	18.3122*	(Target Mode Visually ID: P-107)
41	108	2.0461	18.3412*	
42	116	6.5650	18.8164**	(Target Mode Visually ID: S-116)
43	117	3.1380	18.9689*	
44	152	1.5735	20.9458	
45	154	1.5635	21.0687	
46	157	1.5186	21.2594	
47	160	1.9423	21.3991	
48	164	1.6495	21.6620	
49	212	1.6487	24.0730	

Table 2

50	261	2.5674	26.1254*	
51	274	1.5853	26.9555	(Target Mode Visually ID: P-274)
--	277	---	27.13	(Missed' P-277)
--	279	---	27.31	(Missed' P-279)
--	282	---	27.45	(Missed' P-282)
52	329	2.4262	28.9973*	
--	407	---	32.08	(Missed' S-407)
--	477	---	34.33	(Missed' P-477)
--	485	---	34.62	(Missed' P-485)
53	610	2.6123	40.5606*	
54	613	1.7955	40.8132	
55	628	1.7286	41.6292	(Target Mode Visually ID: S-628)
56	727	1.6035	46.4671	
--	836	---	51.60	(Missed' P-836)
57	839	1.5118	51.7112	
--	840	---	51.81	(Missed' P-840)
58	870	2.0894	53.0968*	
59	875	1.6243	53.4129	

' Aero-surface modes: RSS ran for individual components

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Forward to Aft Motion (x-dir)

	mode #	$\Sigma R_x$ Value	Freq. (Hz)	
1	1	7.1223	0.1808**	
2	2	13.4619	0.3646**	
3	4	82.8039	1.3741**	(Vehicle Axial Mode : S-4)
4	7	4.3747	2.1408**	
5	8	6.0292	2.2473**	
6	9	7.2702	2.3667**	
7	18	4.2681	7.7719**	
8	19	2.1477	7.8355*	
9	23	2.3386	8.8501*	
10	24	1.5789	9.2583	
11	42	9.5193	11.6068**	(Vehicle POGO Axial Mode : P-42)
12	45	4.5880	12.2860**	
13	82	2.6974	15.4933*	
14	97	1.7327	17.2026	
15	101	1.8612	17.7632	
16	116	2.8103	18.8164*	
17	261	1.5547	26.1254	

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Port to Star-port Motion (y-dir)

	mode #	$\Sigma R_y$ Value	Freq. (Hz)	
1	2	24.0329	0.3646**	(Vehicle Yaw Mode : S-2)
2	3	2.2410	0.5696*	
3	5	3.6776	1.4554**	
4	6	10.4107	1.4914**	(Vehicle Roll y-z Mode : S-6)
5	7	11.8624	2.1408**	
6	8	17.3874	2.2473**	(Vehicle Yaw Mode : S-8)
7	16	1.8746	6.4581	
8	18	2.1179	7.7719*	
9	19	3.9281	7.8355**	(Vehicle Y Mode : P-19)
10	20	1.5890	8.3583	
11	22	1.5096	8.5490	
12	23	3.5030	8.8501**	
13	24	2.2186	9.2583*	
14	40	2.0427	11.3322*	
15	42	3.0362	11.6068*	
16	45	1.5447	12.2860	
17	82	1.9589	15.4933	
18	92	1.7302	16.7441	

Table 2 (con't)



19	94	2.1057	16.9149*
20	96	1.9079	17.0105
21	97	1.5957	17.2026
22	99	2.0010	17.3587*
23	101	2.0222	17.7632*
24	116	1.9025	18.8164
25	117	1.5211	18.9689

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Top to Bottom Motion (z-dir)			
	mode #	$\Sigma R_z$ Value	Freq. (Hz)
1	1	21.0653	0.1808** (Vehicle Pitch Mode : S-1)
2	5	6.7476	1.4554**
3	6	20.7954	1.4914** (Vehicle Roll y-z Mode : S-6)
4	9	41.7740	2.3667** (Vehicle Pitch Mode : S-9)
5	11	1.7361	4.1329
6	13	2.9030	5.5324*
7	15	3.2964	6.3970*
8	16	3.9712	6.4581**
9	18	15.2255	7.7719** (Vehicle Z bending : P-18)
10	19	5.9399	7.8355**
11	24	5.9501	9.2583**
12	30	2.7040	10.3370*
13	32	2.9599	10.5886*
14	35	2.8613	11.0533*
15	40	1.7366	11.3322
16	82	3.5613	15.4933**
17	101	1.5321	17.7632
18	116	4.3574	18.8164**
19	117	1.7109	18.9689
20	329	1.5957	28.9973

Table 2 (con't)